## 2011 IMSA Junior High Math Competition 7<sup>th</sup> Individual

- 1. A cube has surface area of  $m^2$ . What is its volume in  $m^3$ ?
- 2. Alexa runs at a rate of 3 mile per hour. Adriana runs at a rate of 4.5 miles per hour. If Adriana starts running an hour after Alexa, how many hours does it take for her to catch up?
- 3. If 8 cows eat 9 acres of grass in 4 days, how much grass with 2 cows eat in 8 days?
- 4. Joseph has 30 marbles in an urn: 14 are red, 13 are green, 1 is blue, and the rest are yellow. If Joseph draws a marble at random, replaces it, then draws another marble, what is the probability that he will draw a blue and then a red marble?
- 5. A triangle has sides of length 3, 4, and 5. What is its area?
- 6. If  $\frac{3a}{4} = \frac{5b}{6}$ , what is  $\frac{a-b}{a}$ ?
- 7. If a: b = 1: 4 and b: c = 3: 7, what is a: c?
- 8. A box has length 4, width 16, and height 8. How many 2 by 2 by 2 cubes can fit in the box?
- 9. There are 230 sophomores at IMSA who take Spanish and/or French. If 150 take Spanish and 213 take French, how many students are taking both languages?
- 10. There is a room full of cats and birds. If there are 18 legs in a room and there are as many cats and birds, how many cat legs are in the room?
- 11. Anna flips a coin until she gets tails twice in a row and then she stops. If she flipped the coin four times and then stopped, what is the probability that she got heads twice?
- 12. What is the probability that the sum of two dice rolled is an even number less than 6?
- 13. Katie's alarm clock goes off every 10 minutes and her doorbell rings every 13 minutes. If she can only wake up when they both go off at the same time and the alarm clock starts at 8:00 and the doorbell first rings at 8:05, when does Katie wake up?

- 14. If  $\phi(y) = 3 \times (4 + y)$  what is  $\phi(\phi 4) \phi(3)$ ?
- 15. A square is placed inside a circle with each corner of the square on the circle. An equilateral triangle is placed inside the square with its base on the base of the square. If the circle has radius of 8, what is the area of the triangle?
- 16. In how many ways can letters in the word "BIBLIOBULI" be rearranged?
- 17. How many diagonals are in a 23-sided regular polygon?
- 18. How many squares that include the black square can be formed from the 7 by 7 grid?

- 19. What is the greatest prime factor of 21! + 22! (where  $n! = n \cdot (n-1) \cdot (n-2) \cdots 3 \cdot 2 \cdot 1$ )?
- 20. Snickerdoodles come in bags of either 7 or 13. Stanley wants to buy an exact amount of snickerdoodles by buying a combination of 7 and 13 bags. For example, Stanley can buy exactly 47 snickerdoodles by buying 3 bags of 7 and 2 bags of 13; however, there is no way to buy exactly 18 snickerdoodles. What is the greatest number of snickerdoodles that Stanley can't buy?

## 2011 IMSA Junior High Math Competition 8th Individual

- 1. Dr. Condie has a collection of 40 songs that are each 3 minutes in length and 60 songs that are each 5 minutes in length. What is the maximum number of songs from his collection that he can play in 5 hours?
- Vincent and Noah are playing a game where they flip three coins. If all three coins come up the same, Vincent wins. Otherwise, Noah wins. Who has the best chance of winning? For your answer simply write Vincent, Noah, or Same (if they both have the same chance to win.
- 3. If  $x^2 + y^2 = 39$  and  $x^2 y^2 = 15$ , what is the value of |xy|?
- 4. If  $x\$y = x \cdot (x^5 + y^2)$ , what is 2\$5?
- 5. Simplify the expression:  $\left(1-\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1+\frac{1}{5}\right)\cdots\left(1+\frac{1}{2011}\right)\left(1-\frac{1}{2012}\right)$ .
- 6. Find x if  $\sqrt{x \sqrt{x \sqrt{x$
- 7. Ten percent less than 50 is one-eighth more than what number?
- 8. Depending on where parentheses are entered, the expression  $1 \ge 2 + 3 \ge 4$  can have a number of different values. How many different values are there?
- 9. If a line passes through the points  $\left(-\frac{3}{2},1\right)$  and (1, 2), what is its *x*-intercept?
- 10. Webster is thinking of a two-digit number less than 50. If you double his number and subtract 12, you get the original number with the digits reversed! What number is Webster thinking about?
- 11. What is the last digit of  $2^{2011}$ ?

- 12. How many distinct ways are there to arrange the letters IMSAJHMC?
- 13. If 15! is multiplied out, how many zeroes will there be at the end?
- 14. How many ways are there for Dr. Fogel to choose a group of three students to write problems for the JHMC from his Number Theory class of ten students?
- 15. A quadrilateral has all of its vertices on a circle and has side lengths AB=39, BC=52, CD=25, and AD=60. Calculate the area of this quadrilateral.
- 16. Find the number of different routes from point A to point B always heading north or east.



- 17. If it takes Peter 10 minutes total to do his 10 problem sets, each of which has 10 problems, how many minutes will it take him to do all 100 problem sets for his 100 friends if each of their problem sets has 100 problems?
- 18. What is the minimum number of people in a room before there is guaranteed to be at least 3 people who all know each other or at least 4 people who have never met? Assume that given any pair of people in the room, they either both know each other or they have never met.
- 19. How many integers *n* are there such that  $\frac{4n-9}{2n+13}$  is an integer? Note: an integer is a number in the set {..., -3, -2, -1, 0, 1, 2, 3, ...}.
- 20. Derek has 5 t-shirts, 4 sweaters, 3 pairs of jeans, and two pairs of shoes. If he always wears one t-shirt, one sweater, a pair of jeans, and a pair of shoes, how many possible outfits can he wear?

## 2011 IMSA Junior High Math Competition 7<sup>th</sup> Team

- 1. Michelle's 8<sup>th</sup> grade class has 190 students. If there are 6 more boys than there are girls in her class. How many boys are there in her class?
- 2. What is the area of a triangle with coordinates (5,4), (5,7) and (9,4)?
- 3. Alison takes a number adds 500 to it, divides by 3, subtracts 86, and ends up with the same number that she started with. What is that number?
- 4. Haley is running clockwise around an 8 mile long track at 7 miles per hour. Her archenemy Sarah is running counterclockwise around the same track at 6 miles per hour. If they both start at the same time, how many minutes will it take until they meet? Round your answer to the nearest minute.
- 5. How many degrees are in the supplement to the interior angle of a regular nine-sided polygon?
- 6. What is the least common multiple of 52 and 117?
- 7. The area of trapezoid A is 35. If trapezoid A contains a base of 10 and a height of 5, what is the length of the other base?
- 8. What has a greater area: a circle with radius 2 or a square with perimeter 8? Please write Square or Circle as your answer.
- 9. What is the surface area of a cube with main diagonal 15 meters in length? Give your answer in square meters.
- 10. In a standard deck of 52 cards, what is the probability of drawing a red six, replacing it, and then drawing a face card (jack, queen, or king)?

11. If 
$$f\left(\frac{x+2}{x}\right) = x^2 + 6x + 2$$
, then what is  $f(3)$ ?

- 12. Emma has older twin brothers. The product of their three ages is 147. What is the sum of their three ages?
- 13. Dr. Krouse can walk up stairs either one or two steps at a time. Her stepping sequence is not necessarily regular. For example, she might step up two steps, then two again, then one step, then one, then two, and then one more to climb up 9 steps. In how many different ways can Dr. Krouse walk up a 12-step stairwell?
- 14. Within the mathletes at JHMC, 170 students were surveyed on what type of math questions they liked: Probability, Geometry, and Number Theory. The results are listed below:

Probability	107
Geometry	90
Number Theory	71
Probability and Geometry	56
Geometry and Number Theory	40
Probability and Number Theory	47
Probability, Geometry, and Number Theory	36

How many students do not like questions on any of the three topics?

- 15. The scale of a map is 1/16 inch=5 miles. On the map, Chicago and San Francisco are 37.5 inches apart, Chicago and Nashville are 8.75 inches apart, and Nashville and San Francisco are 43.75 inches apart. What is the perimeter, in miles, of the triangle formed by these three cities?
- 16. How many integers x satisfy the following inequality?  $(x - 2006)(x - 2004)(x - 2002) \dots (x - 4) \le 0$

- 17. There are a total of 3993 digits used to number a book. How many pages are in the book?
- 18. How many positive 7-digit odd integers are palindromes? A palindrome is the same when written left to right as it is right to left, for example, 2315132.
- 19. How many six digit numbers have their digits increasing order? For example, 135679.
- 20. Let S be equal to the sum:  $1 + 2 + 3 + \dots + 2011$ . Find the remainder when S is divided by 1000.

## 2011 IMSA Junior High Math Competition 8<sup>th</sup> Team

- An old woman carrying a basket of strawberries dropped her basket accidentally. The berries rolled out, but a stranger stopped and helped her pick them up.
  "How many strawberries did you have?" the stranger asked.
  "I don't quite remember. Well, when I tried to divide them into two baskets, there was one strawberry left over. When I divided them into three baskets, there were two left over, and there were three strawberries left over when I tried to divide them into four baskets. Four strawberries were left over when I divided them into five baskets," she answered. What is the smallest number of berries that she could have started with in her basket?
- 2. Find the smallest perfect square that is a multiple of 140.
- 3. If  $2^n$  is a factor of (230!), what is the largest possible value of n?
- 4. What is the coefficient of the  $x^2y^5$  term in the expansion of  $(-2x + y)^7$ ?
- 5. The greatest common divisor of 360, 900, and x is 90. If 600 < x < 800, what is x?
- 6. It takes Katie 30 minutes to drive from her house to Corinne's house at 65 miles per hour. If Corinne is driving at 40 miles per hour, how many minutes will it take her to get from her house to Katie's house?
- 7. Andrew is randomly drawing from a deck of cards (which contains 52 cards with 13 of each suit) without replacement. What is the probability that Andrew draws the four aces in any order?
- 8. If a salad calls for 2 lettuce leaves per 3 slices of cucumbers, 4 slices of cucumbers per 5 croutons, 6 croutons per 7 cherry tomatoes, and 8 cherry tomatoes per 9 spinach leaves, what is the ratio of lettuce leaves to spinach leaves?
- 9. If *a*, *b*, *c* and *d* are all distinct integers and a > b, c > b, d > a, and d > c, what is the sum of all the possibilities for *c* provided that d is 18, a is 13, and b is 10?
- 10. Out of 50 marbles, there are 10 pink marbles, 14 green marbles, 18 blue marbles, and some number of yellow marbles. Martin wins if he can draw with replacement one marble of each color in any order (so one pink marble, one green, one blue, and one yellow). He loses if he draws a marble that matches the color of one he has already drawn. What is the probability that he loses on his second draw?
- 11. If  $x^2 + y^2 = 40$  and xy = 11, then what does  $(x + y)^2$  equal?

- 12. Find the area of a regular hexagon with sides of length 10 inches.
- 13. Dr. Condie is a coin collecting fanatic. He happens to like the New York and Massachusetts State quarters the best, and has 29 and 11 of them, respectively. All of his New York quarters are kept heads up, while all of his Massachusetts quarters are kept heads down. Dr. Keyton decides to run in and flip over 20 of Dr. Condie's quarters randomly. Find the expected number of Dr. Condie's quarters that are heads up when Dr. Keyton is finished.
- 14. How many integers x satisfy the following inequality?  $(x - 2006)(x - 2004)(x - 2002) \dots (x - 4) \le 0$
- 15. Dr. Prince bought a dozen cookies. He wanted at least one each of M&M, sugar, and walnut. Find the number of different combinations of cookies that he could have bought.
- 16. Find the number of ordered pairs of positive integers (a, b, c, d) that satisfy the following equation:

$$a+b+c+d=13$$

- 17. When a right triangle is rotated about one leg, a cone with volume  $4800\pi$  is formed. When the triangle is rotated about its other leg, a cone with volume  $1080\pi$  is formed. What is the hypotenuse of the triangle?
- 18. What is the smallest integer n such that n! ends in at least 100 zeros?
- 19. Find the last two digits of  $12^{12^{12}}$ .

Three regular, six-sided dice are rolled successively. What is the probability that they are in

strictly decreasing order?